# CHAPTER 15 Matrices and Determinants

## Section-A JEE Advanced/ IIT-JEE

### A Fill in the Blanks

1. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$

be an identity in $\lambda$, where $p$, $q$, $r$, $s$ and $t$ are constants. Then, the value of $t$ is .................. *(1981 - 2 Marks)*

2. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is .................. *(1981 - 2 Marks)*

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of determinant chosen is positive is .................. *(1982 - 2 Marks)*

4. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ the other two roots are ................. and .............. *(1983 - 2 Marks)*

5. The system of equations

$$\lambda x + y + z = 0$$
$$-x + \lambda y + z = 0$$
$$-x - y + \lambda z = 0$$

Will have a non-zero solution if real values of $\lambda$ are given by .................. *(1984 - 2 Marks)*

6. The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is .................. *(1988 - 2 Marks)*

7. For positive numbers $x$, $y$ and $z$, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is .................. *(1993 - 2 Marks)*

### B True/ False

1. The determinants *(1983 - 1 Mark)*

$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal.

2. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

then the two triangles with vertices $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$, and $(a_1, b_1)$, $(a_2, b_2)$, $(a_3, b_3)$ must be congruent. *(1985 - 1 Mark)*

### C MCQs with One Correct Answer

1. Consider the set $A$ of all determinants of order 3 with entries 0 or 1 only. Let $B$ be the subset of $A$ consisting of all determinants with value 1. Let $C$ be the subset of $A$ consisting of all determinants with value $-1$. Then
   (a) $C$ is empty *(1981 - 2 Marks)*
   (b) $B$ has as many elements as $C$
   (c) $A = B \cup C$
   (d) $B$ has twice as many elements as elements as $C$

2. If $\omega (\neq 1)$ is a cube root of unity, then

$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$ *(1995S)*

   (a) 0 (b) 1 (c) i (d) $\omega$

3. Let $a$, $b$, $c$ be the real numbers. Then following system of equations in $x$, $y$ and $z$ *(1995S)*

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$,

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

   (a) no solution (b) unique solution
   (c) infinitely many solutions (d) finitely many solutions

4. If $A$ and $B$ are square matrices of equal degree, then which one is correct among the followings? *(1995S)*
(a) $A+B=B+A$ (b) $A+B=A-B$
(c) $A-B=B-A$ (d) $AB=BA$

5. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is *(1997 - 2 Marks)*
(a) $a$ (b) $p$ (c) $d$ (d) $x$

6. If $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to *(1999 - 2 Marks)*
(a) 0 (b) 1 (c) 100 (d) –100

7. If the system of equations $x-ky-z=0,\ kx-y-z=0,\ x+y-z=0$ has a non-zero solution, then the possible values of $k$ are *(2000S)*
(a) –1, 2 (b) 1, 2 (c) 0, 1 (d) –1, 1

8. Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is *(2002S)*
(a) $3\omega$ (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

9. The number of values of $k$ for which the system of equations $(k+1)x+8y=4k;\ kx+(k+3)y=3k-1$ has infinitely many solutions is *(2002S)*
(a) 0 (b) 1 (c) 2 (d) infinite

10. If $A=\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of $\alpha$ for which $A^2=B$, is *(2003S)*
(a) 1 (b) –1
(c) 4 (d) no real values

11. If the system of equations $x+ay=0,\ az+y=0$ and $ax+z=0$ has infinite solutions, then the value of $a$ is *(2003S)*
(a) –1 (b) 1
(c) 0 (d) no real values

12. Given $2x-y+2z=2,\ x-2y+z=-4,\ x+y+\lambda z=4$ then the value of $\lambda$ such that the given system of equation has NO solution, is *(2004S)*
(a) 3 (b) 1 (c) 0 (d) $-3$

13. If $A=\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3|=125$ then the value of $\alpha$ is *(2004S)*
(a) $\pm1$ (b) $\pm2$ (c) $\pm3$ (d) $\pm5$

14. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1}=\left[\frac{1}{6}(A^2+cA+dI)\right]$, then the value of $c$ and $d$ are *(2005S)*
(a) $(-6,-11)$ (b) $(6,11)$ (c) $(-6,11)$ (d) $(6,-11)$

15. If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q=PAP^T$ and $x=P^TQ^{2005}P$ then $x$ is equal to *(2005S)*
(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$
(c) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$
(d) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

16. Consider three points $P=(-\sin(\beta-\alpha),-\cos\beta)$, $Q=(\cos(\beta-\alpha),\sin\beta)$ and $R=(\cos(\beta-\alpha+\theta),\sin(\beta-\theta))$, where $0<\alpha,\beta,\theta<\frac{\pi}{4}$. Then, *(2008)*
(a) $P$ lies on the line segment $RQ$
(b) $Q$ lies on the line segment $PR$
(c) $R$ lies on the line segment $QP$
(d) $P, Q, R$ are non-collinear

17. The number of $3\times 3$ matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is *(2010)*
(a) 0 (b) $2^9-1$ (c) 168 (d) 2

**18.** Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of *a, b* and *c* is either $\omega$ or $\omega^2$. Then the number of distinct matrices in the set S is *(2011)*

(a) 2 (b) 6 (c) 4 (d) 8

**19.** Let $P = [a_{ij}]$ be a $3 \times 3$ matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of $P$ is 2, then the determinant of the matrix $Q$ is *(2012)*

(a) $2^{10}$ (b) $2^{11}$ (c) $2^{12}$ (d) $2^{13}$

**20.** If $P$ is a $3 \times 3$ matrix such that $P^T = 2P + I$, where $P^T$ is the transpose of $P$ and $I$ is the $3 \times 3$ identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that *(2012)*

(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $PX = X$

(c) $PX = 2X$ (d) $PX = -X$

**21.** Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and $I$ be the identity matrix of order 3.

If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals *(JEE Adv. 2016)*

(a) 52 (b) 103

(c) 201 (d) 205

## D MCQs with One or More than One Correct

**1.** The determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if *(1986 - 2 Marks)*

(a) *a, b, c* are in A. P.

(b) *a, b, c* are in G. P.

(c) *a, b, c* are in H. P.

(d) $\alpha$ is a root of the equation $ax^2 + bx + c = 0$

(e) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$.

**2.** If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then *(1998 - 2 Marks)*

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$

(c) $x = 0, y = 3$ (d) $x = 0, y = 0$

**3.** Let M and N be two $3 \times 3$ non-singular skew- symmetric matrices such that MN = NM. If $P^T$ denotes the transpose of P, then $M^2N^2 (M^TN)^{-1} (MN^{-1})^T$ is equal to *(2011)*

(a) $M^2$ (b) $-N^2$ (c) $-M^2$ (d) MN

**4.** If the adjoint of a $3 \times 3$ matrix $P$ is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of $P$ is (are) *(2012 )*

(a) –2 (b) –1 (c) 1 (d) 2

**5.** For $3 \times 3$ matrices M and N, which of the following statement(s) is (are) **NOT** correct? *(JEE Adv. 2013)*

(a) $N^TMN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(b) MN – NM is skew symmetric for all symmetric matrices M and N

(c) MN is symmetric for all symmetric matrices M and N

(d) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

**6.** Let $\omega$ be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when n = *(JEE Adv. 2013)*

(a) 57 (b) 55 (c) 58 (d) 56

**7.** Let $M$ be a $2 \times 2$ symmetric matrix with integer entries. Then $M$ is invertible if *(JEE Adv. 2014)*

(a) The first column of $M$ is the transpose of the second row of $M$

(b) The second row of $M$ is the transpose of the first column of $M$

(c) $M$ is a diagonal matrix with non-zero entries in the main diagonal

(d) The product of entries in the main diagonal of $M$ is not the square of an integer

**8.** Let $M$ and $N$ be two $3 \times 3$ matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then *(JEE Adv. 2014)*

(a) determinant of $(M^2 + MN^2)$ is 0

(b) there is $3 \times 3$ non-zero matrix $U$ such that $(M^2 + MN^2)U$ is the zero matrix

(c) determinant of $(M^2 + MN^2) \ge 1$

(d) for a $3 \times 3$ matrix $U$, if $(M^2 + MN^2)U$ equals the zero matrix then $U$ is the zero matrix

**9.** Which of the following values of $\alpha$ satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha ?$$

*(JEE Adv. 2015)*

(a) –4 (b) 9 (c) –9 (d) 4

**10.** Let $X$ and $Y$ be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and $Z$ be an arbitrary 3 × 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? *(JEE Adv. 2015)*

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

**11.** Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then *(JEE Adv. 2016)*

(a) $a = 0, k = 8$ (b) $4a - k + 8 = 0$

(c) $\det(P \operatorname{adj}(Q)) = 2^9$ (d) $\det(Q \operatorname{adj}(P)) = 2^{13}$

**12.** Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is (are) correct? *(JEE Adv. 2016)*

(a) If $a = -3$, then the system has infinitely many solutions for all values of $\lambda$ and $\mu$.

(b) If $a \neq -3$, then the system has a unique solution for all values of $\lambda$ and $\mu$.

(c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.

(d) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$.

## E Subjective Problems

**1.** For what value of $k$ do the following system of equations possess a non trivial (i.e., not all zero) solution over the set of rationals $Q$?

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 3y - 4z = 0$

For that value of $k$, find all the solutions for the system. *(1979)*

**2.** Let $a, b, c$ be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative. *(1981 - 4 Marks)*

**3.** Without expanding a determinant at any stage, show that

$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B,$$ where $A$ and $B$ are determinants of order 3 not involving $x$. *(1982 - 5 Marks)*

**4.** Show that

$$\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

*(1985 - 2 Marks)*

**5.** Consider the system of linear equations in $x, y, z$:

$(\sin 3\theta)\, x - y + z = 0$

$(\cos 2\theta)\, x + 4y + 3z = 0$

$2x + 7y + 7z = 0$

Find the values of $\theta$ for which this system has nontrivial solutions. *(1986 - 5 Marks)*

**6.** Let $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$.

Show that $\sum_{a=1}^{n} \Delta a = c$, a constant. *(1989 - 5 Marks)*

**7.** Let the three digit numbers $A$ 28, 3$B$9, and 62 $C$, where $A, B,$ and $C$ are integers between 0 and 9, be divisible by a fixed integer $k$. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by $k$. *(1990 - 4 Marks)*

**8.** If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ *(1991 - 4 Marks)*

**9.** For a fixed positive integer $n$, if *(1992 - 4 Marks)*

$$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$

then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by $n$.

**10.** Let $\lambda$ and $\alpha$ be real. Find the set of all values of $\lambda$ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$, $x + (\cos\alpha)y + (\sin\alpha)z = 0$,

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution. For $\lambda = 1$, find all values of $\alpha$.

*(1993 - 5 Marks)*

**11.** For all values of $A, B, C$ and $P, Q, R$ show that

*(1994 - 4 Marks)*

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

**12.** Let $a > 0,\ d > 0$. Find the value of the determinant

*(1996 - 5 Marks)*

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

**13.** Prove that for all values of $\theta$,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

*(2000 - 3 Marks)*

**14.** If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where $a$, $b$, $c$ are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. *(2003 - 2 Marks)*

**15.** If $M$ is a $3 \times 3$ matrix, where det $M = 1$ and $MM^T = I$, where 'I' is an identity matrix, prove that det $(M - I) = 0$.

*(2004 - 2 Marks)*

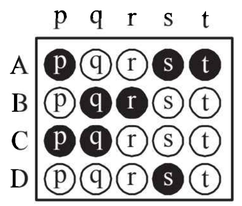
**16.** If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $AX = U$ has infinitely many solutions, prove that $BX = V$ has no unique solution. Also show that if afd $\neq$ 0, then $BX = V$ has no solution. *(2004 - 4 Marks)*

## F Match the Following

*Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

*If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.*




**1.** Consider the lines given by

$L_1 : x + 3y - 5 = 0; L_2 : 3x - ky - 1 = 0; L_3 : 5x + 2y - 12 = 0$

Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the $4 \times 4$ matrix given in the ORS. *(2008)*

| ***Column I*** | ***Column II*** |
|---|---|
| (A) $L_1, L_2, L_3$ are concurrent, if | (p) $k = -9$ |
| (B) One of $L_1, L_2, L_3$ is parallel to at least one of the other two, if | (q) $k = -\frac{6}{5}$ |
| (C) $L_1, L_2, L_3$ from a triangle, if | (r) $k = \frac{5}{6}$ |
| (D) $L_1, L_2, L_3$ do not form a triangle, if | (s) $k = 5$ |

2. Match the Statements/Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the $4 \times 4$ matrix given in the ORS. ***(2008)***

| *Column I* | *Column II* |
|---|---|
| (A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is | (p) 0 |
| (B) Let A and B be $3 \times 3$ matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t=(-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of $k$ are | (q) 1 |
| (C) Let $a=\log_3 \log_3 2$. An integer $k$ satisfying $1<2^{(-k+3^{-a})}<2$, must be less than | (r) 2 |
| (D) If $\sin\theta=\cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are | (s) 3 |

## G Comprehension Based Questions

### PASSAGE - 1

Let $A=\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}$, and $U_1$, $U_2$ and $U_3$ are columns of a $3 \times 3$ matrix $U$. If column matrices $U_1$, $U_2$ and $U_3$ satisfying $AU_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, AU_2=\begin{bmatrix}2\\3\\0\end{bmatrix}, AU_3=\begin{bmatrix}2\\3\\1\end{bmatrix}$ evaluate as directed in the following questions.

1. The value $|U|$ is ***(2006 - 5M, –2)***
   (a) 3 (b) –3 (c) $\frac{3}{2}$ (d) 2
2. The sum of the elements of the matrix $U^{-1}$ is ***(2006 - 5M, –2)***
   (a) –1 (b) 0 (c) 1 (d) 3
3. The value of $[3\ 2\ 0]\, U \begin{bmatrix}3\\2\\0\end{bmatrix}$ is ***(2006 - 5M, –2)***
   (a) 5 (b) $\frac{5}{2}$ (c) 4 (d) $\frac{3}{2}$

### PASSAGE - 2

Let $\mathscr{A}$ be the set of all $3\times 3$ symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

4. The number of matrices in $\mathscr{A}$ is ***(2009)***
   (a) 12 (b) 6 (c) 9 (d) 3
5. The number of matrices $A$ in $\mathscr{A}$ for which the system of linear equations
$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$
has a unique solution, is ***(2009)***
   (a) less than 4
   (b) at least 4 but less than 7
   (c) at least 7 but less than 10
   (d) at least 10
6. The number of matrices $A$ in $\mathscr{A}$ for which the system of linear equations
$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$
is inconsistent, is ***(2009)***
   (a) 0 (b) more than 2
   (c) 2 (d) 1

### PASSAGE - 3

Let $p$ be an odd prime number and $T_p$ be the following set of $2 \times 2$ matrices :

$$T_P=\left\{A=\begin{bmatrix}a&b\\c&a\end{bmatrix}: a,b,c \in \{0,1,2,\ldots,p-1\}\right\}$$ ***(2010)***

7. The number of A in $T_p$ such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is
   (a) $(p-1)^2$ (b) $2(p-1)$
   (c) $(p-1)^2+1$ (d) $2p-1$
8. The number of A in $T_p$ such that the trace of A is not divisible by p but det (A) is divisible by p is
   [**Note:** The trace of a matrix is the sum of its diagonal entries.]
   (a) $(p-1)(p^2-p+1)$ (b) $p^3-(p-1)^2$
   (c) $(p-1)^2$ (d) $(p-1)(p^2-2)$
9. The number of A in $T_p$ such that det (A) is not divisible by p is
   (a) $2p^2$ (b) $p^3-5p$
   (c) $p^3-3p$ (d) $p^3-p^2$

**PASSAGE - 4**

Let $a$, $b$ and $c$ be three real numbers satisfying **(2011)**

$$[a\,b\,c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0\,0\,0] \quad \text{...(E)}$$

**10.** If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is
(a) 0 (b) 12 (c) 7 (d) 6

**11.** Let $\omega$ be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$, if $a = 2$ with $b$ and $c$ satisfying (E), then the value of $\dfrac{3}{\omega^a}+\dfrac{1}{\omega^b}+\dfrac{3}{\omega^c}$ is equal to
(a) $-2$ (b) 2 (c) 3 (d) $-3$

**12.** Let $b = 6$, with $a$ and $c$ satisfying (E). If $\alpha$ and $\beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n \text{ is}$$

(a) 6 (b) 7 (c) $\dfrac{6}{7}$ (d) $\infty$

## H Assertion & Reason Type Questions

**1.** Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1$$

**STATEMENT - 1 :** The system of equations has no solution for $k \neq 3$ and

**STATEMENT-2 :** The determinant $\begin{vmatrix}1 & 3 & -1\\ -1 & -2 & k\\ 1 & 4 & 1\end{vmatrix} \neq 0$, for $k \neq 3$. **(2008)**

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explaination for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

## I Integer Value Correct Type

**1.** Let $\omega$ be the complex number $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$. Then the number of distinct complex numbers $z$ satisfying

$$\begin{vmatrix}z+1 & \omega & \omega^2\\ \omega & z+\omega^2 & 1\\ \omega^2 & 1 & z+\omega\end{vmatrix}=0 \text{ is equal to}$$ **(2010)**

**2.** Let $k$ be a positive real number and let

$$A=\begin{bmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{bmatrix} \text{ and } B=\begin{bmatrix}0 & 2k-1 & \sqrt{k}\\ 1-2k & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{bmatrix}$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$. then [k] is equal to
[**Note** : adj M denotes the adjoint of square matrix $M$ and $[k]$ denotes the largest integer less than or equal $k$. **(2010)**

**3.** Let M be a 3 × 3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$. Then the sum of the diagonal entries of $M$ is **(2011)**

**4.** The total number of distinct $x \in \mathbb{R}$ for which

$$\begin{vmatrix}x & x^2 & 1+x^3\\ 2x & 4x^2 & 1+8x^3\\ 3x & 9x^2 & 1+27x^3\end{vmatrix}=10 \text{ is}$$ ***(JEE Adv. 2016)***

**5.** Let $z=\dfrac{-1+\sqrt{3}i}{2}$, where $i=\sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let

$$P=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$$ and I be the identity matrix of order 2.

Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is ***(JEE Adv. 2016)***

## Section-B JEE Main / AIEEE

1. If $a > 0$ and discriminant of $ax^2+2bx+c$ is –ve, then

$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} \text{ is equal to}$$ **[2002]**

(a) +ve (b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve (d) 0

2. If the system of linear equations **[2003]**
$x+2ay+az=0$ ; $x+3by+bz=0$ ; $x+4cy+cz=0$ ;
has a non - zero solution, then a, b, c.
(a) satisfy $a+2b+3c=0$ (b) are in A.P
(c) are in G.P (d) are in H.P.

3. If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} \text{ is equal to}$$ **[2003[**

(a) $\omega^2$ (b) 0 (c) 1 (d) $\omega$

4. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[2003[**

(a) $\alpha = 2ab, \beta = a^2+b^2$
(b) $\alpha = a^2+b^2,\ \beta = ab$
(c) $\alpha = a^2+b^2,\ \beta = 2ab$
(d) $\alpha = a^2+b^2,\ \beta = a^2-b^2$.

5. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix $A$ is **[2004]**
(a) $A^2 = I$
(b) $A = (-1)\,I$, where $I$ is a unit matrix
(c) $A^{-1}$ does not exist
(d) $A$ is a zero matrix

6. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$. and $10\,B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If $B$ is the inverseof matrix $A$, then $\alpha$ is **[2004]**
(a) 5 (b) –1 (c) 2 (d) –2

7. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots.$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

(a) –2 (b) 1 (c) 2 (d) 0

8. If $A^2 - A + I = 0$, then the inverse of $A$ is **[2005]**
(a) $A+I$ (b) $A$ (c) $A-I$ (d) $I-A$

9. The system of equations
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$
has infinite solutions, if $\alpha$ is **[2005]**
(a) $-2$ (b) either $-2$ or 1
(c) not $-2$ (d) 1

10. If $a^2+b^2+c^2 = -2$ and **[2005]**

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix},$$

then f(x) is a polynomial of degree
(a) 1 (b) 0 (c) 3 (d) 2

11. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n, \ldots\ldots$ are in G P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to **[2005]**
(a) 1 (b) 0
(c) 4 (d) 2

12. If $A$ and $B$ are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then which of the following will be always true? **[2006]**
(a) $A = B$
(b) $AB = BA$
(c) either of A or B is a zero matrix
(d) either of A or B is identity matrix

13. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then **[2006]**
(a) there cannot exist any B such that AB = BA
(b) there exist more then one but finite number of B′s such that AB = BA

(c) there exists exactly one B such that AB = BA
(d) there exist infinitely many B′s such that AB = BA

**14.** If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is
(a) divisible by $x$ but not $y$ **[2007]**
(b) divisible by $y$ but not $x$
(c) divisible by neither $x$ nor $y$
(d) divisible by both $x$ and $y$

**15.** Let $A=\begin{vmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{vmatrix}$. If $\left|A^2\right|=25$, then $|\alpha|$ equals **[2007]**
(a) 1/5 (b) 5 (c) $5^2$ (d) 1

**16.** Let $A$ be $a$ 2 × 2 matrix with real entries. Let $I$ be the 2 × 2 identity matrix. Denote by tr($A$), the sum of diagonal entries of $a$. Assume that $A^2 = I$. **[2008]**
**Statement-1 :** If $A \neq I$ and $A \neq -I$, then det($A$) = –1
**Statement-2 :** If $A \neq I$ and $A \neq -I$, then tr ($A$) ≠ 0.
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

**17.** Let $a, b, c$ be any real numbers. Suppose that there are real numbers $x, y, z$ not all zero such that $x = cy + bz, y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[2008]**
(a) 2 (b) –1 (c) 0 (d) 1

**18.** Let $A$ be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**
(a) If det $A = \pm 1$, then $A^{-1}$ exists but all its entries are not necessarily integers
(b) If det $A \neq \pm 1$, then $A^{-1}$ exists and all its entries are non integers
(c) If det $A = \pm 1$, then $A^{-1}$ exists but all its entries are integers
(d) If det $A = \pm 1$, then $A^{-1}$ need not exists

**19.** Let A be a 2 × 2 matrix
**Statement -1 :** adj (adj A) = A
**Statement -2 :** |adj A| = |A| **[2009]**
(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

**20.** Let $a, b, c$ be such that $b(a + c) \neq 0$ if **[2009]**

$$\begin{vmatrix}a & a+1 & a-1\\ -b & b+1 & b-1\\ c & c-1 & c+1\end{vmatrix}+\begin{vmatrix}a+1 & b+1 & c-1\\ a-1 & b-1 & c+1\\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c\end{vmatrix}=0,$$

then the value of $n$ is :
(a) any even integer (b) any odd integer
(c) any integer (d) zero

**21.** The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[2010]**
(a) 5 (b) 6
(c) at least 7 (d) less than 4

**22.** Let $A$ be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where $I$ is 2 × 2 identity matrix. Define
Tr($A$) = sum of diagonal elements of $A$ and
$|A|$ = determinant of matrix $A$.
**Statement - 1 :** Tr($A$) = 0.
**Statement -2 :** $|A|$ = 1. **[2010]**
(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

**23.** Consider the system of linear equations ; **[2010]**
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(a) exactly 3 solutions (b) a unique solution
(c) no solution (d) infinite number of solutions

**24.** The number of values of $k$ for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**
(a) 2 (b) 1 (c) zero (d) 3

**25.** Let $A$ and $B$ be two symmetric matrices of order 3.
**Statement-1:** $A(BA)$ and $(AB)A$ are symmetric matrices.
**Statement-2:** $AB$ is symmetric matrix if matrix multiplication of $A$ with $B$ is commutative. **[2011]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

**26.** Let $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{pmatrix}$. If $u_1$ and $u_2$ are column matrices such that $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$, then $u_1 + u_2$ is equal to : **[2012]**
(a) $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ (b) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ (c) $\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ (d) $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$

**27.** Let $P$ and $Q$ be 3 × 3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : **[2012]**
(a) –2 (b) 1 (c) 0 (d) –1

**28.** If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to : **[JEE M 2013]**

(a) 4 (b) 11 (c) 5 (d) 0

**29.** If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then $K$ is equal to: **[JEE M 2014]**

(a) 1 (b) −1 (c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

**30.** If $A$ is a 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: **[JEE M 2014]**

(a) $B^{-1}$ (b) $(B^{-1})'$ (c) $I + B$ (d) $I$

**31.** The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$
$2x_1 - 3x_2 + 2x_3 = \lambda x_2$
$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution **[JEE M 2015]**

(a) contains two elements
(b) contains more than two elements
(c) is an empty set
(d) is a singleton

**32.** If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to: **[JEE M 2015]**

(a) (2, 1) (b) (−2, − 1)
(c) (2, − 1) (d) (−2, 1)

**33.** The system of linear equations

$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$

has a non-trivial solution for: **[JEE M 2016]**

(a) exactly two values of λ .
(b) exactly three values of λ .
(c) infinitely many values of λ .
(d) exactly one value of λ .

**34.** If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A $A^T$, then 5a + b is equal to : **[JEE M 2016]**

(a) 4 (b) 13
(c) −1 (d) 5

# 15 Matrices and Determinants

## Section-A : JEE Advanced/ IIT-JEE

**A** 1. $t=0$ 2. $x=-1, 2$ 3. 3/16 4. 2, 7 5. $\lambda=0$ 6. 0 7. 0

**B** 1. F 2. F

**C** 1. (b) 2. (a) 3. (d) 4. (a) 5. (b) 6. (a) 7. (d)
8. (b) 9. (b) 10. (d) 11. (a) 12. (b) 13. (c) 14. (c)
15. (a) 16. (d) 17. (a) 18. (a) 19. (d) 20. (d) 21. (b)

**D** 1. (b, e) 2. (d) 3. (c) 4. (a, d) 5. (c, d) 6. (b, c, d) 7. (c, d)
8. (a, b) 9. (b, c) 10. (c, d) 11. (b, c) 12. (b, c, d)

**E** 1. $x=b,\ y=\frac{-2b}{15},\ z=\frac{2b}{5},\ b\in R$ 5. $n\pi$ or $n\pi+(-1)^n\pi/6,\ n\in Z$

8. 2 10. $n\pi$ or $n\pi+\pi/4$

12. $\frac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$ 14. 4

**F** 1. (A) → s; (B) → p, q; (C) → r; (D) → p, q, s 2. (A) → r; (B) → q, s; (C) → r, s; (D) → p, r

**G** 1. (d) 2. (b) 3. (a) 4. (a) 5. (b) 6. (b) 7. (d) 8. (c) 9. (d) 10. (d) 11. (a)
12. (b)

**H** 1. (a)

**I** 1. 0 2. 4 3. 9 4. 2 5. 1

## Section-B : JEE Main/ AIEEE

1. (c) 2. (d) 3. (b) 4. (c) 5. (a) 6. (a) 7. (d)
8. (d) 9. (a) 10. (d) 11. (b) 12. (b) 13. (d) 14. (d)
15. (a) 16. (d) 17. (d) 18. (c) 19. (d) 20. (b) 21. (c)
22. (b) 23. (c) 24. (a) 25. (a) 26. (d) 27. (c) 28. (b)
29. (a) 30. (d) 31. (a) 32. (b) 33. (b) 34. (d)

## Section-A JEE Advanced/ IIT-JEE

### A. Fill in the Blanks

**1.** As given equation is an identity in $\lambda$, it must be true for all values of $\lambda$.

$\therefore$ For $\lambda=0$ also. Putting $\lambda=0$ we get $t=\begin{vmatrix}0 & -1 & 3\\ 1 & 0 & -4\\ -3 & 4 & 0\end{vmatrix}=0$

**2.** Given equation is, $\begin{vmatrix}1 & 4 & 20\\ 1 & -2 & 5\\ 1 & 2x & 5x^2\end{vmatrix}=0$

Clearly on expanding the det. we will get a quadratic equation in $x$.

$\therefore$ It has 2 roots. We observe that $R_3$ becomes identical to $R_1$ if $x=2$. thus at $x=2\Rightarrow\Delta=0$

$\therefore$ $x=2$ is a root of given eq.

Similarly $R_3$ becomes identical to $R_2$ if $x=-1$.

thus at $x=-1\Rightarrow\Delta=0$

$\therefore$ $x=-1$ is a root of given eq.

Hence equation has roots as –1 and 2.

**3.** With 0 and 1 as elements there are 2 × 2 × 2 × 2 = 16 determinants of order 2 × 2 out of which only

$\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix},\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix},\begin{vmatrix}1 & 0\\ 1 & 1\end{vmatrix}$ are the three det whose value is +ve.

$\therefore$ Req. prob. = 3/16

4. $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Operating $R_1 \to R_1 + R_2 + R_3$ we get

$$\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Operating $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & -1 & x-7 \end{vmatrix} = 0$$

Expanding along $R_1$

$\Rightarrow (x+9)(x-2)(x-7) = 0$

$\Rightarrow x = -9, 2, 7$

$\therefore$ Other roots are 2 and 7.

5. The given homogeneous system of equations will have non zero solution if $D = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ -1 & \lambda & 1 \\ -1 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda^2+1) - 1(-\lambda+1) + 1(1+\lambda) = 0 \Rightarrow \lambda^3 + 3\lambda = 0$

$\Rightarrow \lambda(\lambda^2+3) = 0$, but $\lambda^2 + 3 \neq 0$ for real $\lambda \Rightarrow \lambda = 0$

6. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

Operating $R_1 \to R_1 - R_2;\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= (a-b(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

7. Given $x, y, z$ and + ve numbers, then value of

$$D = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix} \qquad \left(\because \log_b a = \frac{\log a}{\log b}\right)$$

Taking $\frac{1}{\log x}, \frac{1}{\log y}$, and $\frac{1}{\log z}$ common from $R_1, R_2$ and $R_3$ respectively

$$D = \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

## B. True/False

1. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = (-1)^2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

[$C_1 \Leftrightarrow C_3$ and then $C_2 \Leftrightarrow C_3$]

$\therefore$ Equal. Hence statement is $F$.

2. (i) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

$Ar(\Delta_1) = Ar(\Delta_2)$

Where $\Delta_1$ is the area of triangle with vertices $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$; and $\Delta_2$ is the area of triangle with ; vertices $(a_1, b_1), (a_2, b_2)$ and $(a_3, b_3)$. But two $\Delta$'s of same area may not be congruent.

$\therefore$ Given statement is false.

## C. MCQs with ONE Correct Answer

1. **(b)** For every 'det, with value 1' ($\in B$) we can find a det. with value – 1 by changing the sign of one entry of '1'. Hence there are equal number of elements in B and C.
$\therefore$ (b) is the correct option

2. **(a)** $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$

Operating $R_1 \to R_1 - R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega+1 & -1 \end{vmatrix} \quad (\because 1+\omega+\omega^2=0)$$
$= 0$

**3. (d)** Let $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y, \frac{z^2}{c^2} = Z$

Then the given system of equations becomes
$X + Y - Z = 1$
$X - Y + Z = 1, \quad -X + Y + Z = 1$
This is the new system of equations
For new system, we have

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1+1) - 1(1-1)$$
$= -4 \neq 0$
$\therefore$ New system of equations has unique solution.

$$D_1 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1-1) - 1(1+1) = -4$$

$$D_2 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(1-1) - 1(1+1) - 1(1+1) = -4$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1+1) + 1(1-1) = -4$$

Now, $X = \frac{D_1}{D} = \frac{-4}{-4} = 1, \; Y = \frac{D_2}{D} = \frac{-4}{-4} = 1$

$Z = \frac{D_3}{D} = \frac{-4}{-4} = 1 \Rightarrow x = \pm a, \; y = \pm b, \; z = \pm c$

**4. (a)** If $A$ and $B$ are square matrices of same degree then matrices $A$ and $B$ can be added or subtracted or multiplied. By algebra of matrices the only correct option is $A + B = B + A$

**5. (b)** Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix}$$

$C_1 \to C_1 - (2\cos dx)C_2$

$$\Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

Expanding along $C_1$, we get
$\Delta = (1 + a^2 - 2a\cos dx)\,[\sin(p+d)x \cos px - \sin px \cos(p+d)x]$
$\Rightarrow \Delta = (1 + a^2 - 2a\cos dx)\,[\sin\{(p+d)x - px\}]$
$\Rightarrow \Delta = (1 + a^2 - 2a\cos dx)\,[\sin dx]$
which is independent of $p$.

**6. (a)**

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

$$= \begin{vmatrix} x+1 & x & x+1 \\ (x+1)x & x(x-1) & (x+1)x \\ (x+1)x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$
$[C_1 \to C_1 + C_2]$

$= 0 \quad [\because C_1$ and $C_2$ are identical$]$
which is free of $x$, so the function is true for all values of $x$. Therefore, at $x = 100, f(x) = 0$, i.e., $f(100) = 0$

**7. (d)** For the given homogeneous system to have non zero solution determinant of coefficient matrix should be zero; i.e.,

$$= \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 1(1+1) + k(-k+1) - 1(k+1) = 0$$
$\Rightarrow 2 - k^2 + k - k - 1 = 0 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

**8. (b)** Given that $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \; \omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$
Also $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$
Now given det. is

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$
[Using $\omega = -1 - \omega^2$ and $\omega^3 = 1$]

Operating $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} \quad (\text{as } 1 + \omega + \omega^2 = 0)$$

Expanding along $C_1$, we get
$3(\omega^2 - \omega^4) = 3(\omega^2 - \omega) = 3\omega(\omega - 1)$

**9. (b)** For infinitely many solutions the two equations become identical

$$\Rightarrow \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1} \Rightarrow k = 1$$

**10. (d)** Given that $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

and $A^2 = B$

$$\Rightarrow \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 5 \Rightarrow \alpha = \pm 1$ and $\alpha = 4$
$\because$ There is no common value
$\therefore$ There is no real value of $\alpha$ for which $A^2 = B$

**11.** **(a)** The given system is, $x + ay = 0$

$az + y = 0$

$ax + z = 0$

It is system of homogeneous equations therefore, it will have infinite many solutions if determinant of coefficient matrix is zero. i.e.,

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1-0) - a(0-a^2) = 0 \Rightarrow 1 + a^3 = 0$

$\Rightarrow a^3 = -1 \Rightarrow a = -1$

**12.** **(b)** Since the system has no solution, $\Delta = 0$ and any one amongst $\Delta_x, \Delta_y, \Delta_z$ is non-zero.

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1$$

$$\text{Also, } \Delta_z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & 4 \end{vmatrix} = 6 \neq 0$$

**13.** **(c)** $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125 \Rightarrow |A|^3 = 125$

Now, $|A| = \alpha^2 - 4$

$\Rightarrow (\alpha^2 - 4)^3 = 125 = 5^3 \Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$

**14.** **(c)** Given $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$

$\therefore$ Characteristic eqn of above matrix $A$ is given by

$$|A - \lambda I| = 0 \Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{bmatrix} = 0$$

$\Rightarrow (1-\lambda)(4 - 5\lambda + \lambda^2 + 2) = 0 \Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Also by Cayley Hamilton thm (every square matrix satisfies its characteristic equation) we obtain

$A^3 - 6A^2 + 11A - 6I = 0$

Multiplying by $A^{-1}$, we get

$$A^2 - 6A + 11I - 6A^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{6}(A^2 - 6A + 11I)$$

Comparing it with given relation,

$$A^{-1} = \frac{1}{6}(A^2 - cA + dI)$$

we get $c = -6$ and $d = 11$

**15.** **(a)** Given that, $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^T Q^{2005} P$

We observe that $Q = PAP^T$

$\Rightarrow Q^2 = (PAP^T)(PAP^T)$

$= PA(P^TP)AP^T = PA(IA)P^T$

$\therefore PA^2P^T$

Proceeding in the same way, we get

$Q^{2005} = PA^{2005}P^T$

Also $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

and proceeding in the same way $A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Now, $X = P^T Q^{2005} P$

$= P^T(PA^{2005}P^T)P = (P^TP)A^{2005}(P^TP)$

$= IA^{2005}I = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

**16.** **(d)** The given points are $P(-\sin(\beta-\alpha), -\cos\beta)$,

$Q(\cos(\beta-\alpha), \sin\beta)$

$R(\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$

Where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$$\therefore \Delta = \begin{vmatrix} 1 & 1 & 1 \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) & \cos(\beta-\alpha+\theta) \\ -\cos\beta & \sin\beta & \sin(\beta-\theta) \end{vmatrix}$$

Operating $C_3 - C_1\sin\theta - C_2\cos\theta$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1-\sin\theta-\cos\theta \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) & 0 \\ -\cos\beta & \sin\beta & 0 \end{vmatrix}$$

$= (1 - \sin\theta - \cos\theta)[\cos\beta\cos(\beta-\alpha) - \sin\beta\sin(\beta-\alpha)]$

$\Rightarrow \Delta = [1 - (\sin\theta + \cos\theta)]\cos(2\beta - \alpha)$

$\because 0 < \alpha, \beta, \theta < \frac{\pi}{4}$ $\quad \therefore \sin\theta + \cos\theta \neq 1$

Also $2\beta - \alpha < \frac{\pi}{4} \Rightarrow \cos(2\beta - \alpha) \neq 0$

$\therefore \Delta \neq 0 \Rightarrow$ the three points are non collinear.

**17.** **(a)** Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ where $a_i, b_i, c_i$ have values 0 or 1 for $i = 1, 2, 3$.

Then the given system is equivalent to

$a_1x + b_1y + c_1z = 1,$

$a_2x + b_2y + c_2z = 0,$

$a_3x + b_3y + c_3z = 0$

Which represents three distinct planes. But three planes can not intersect at two distinct points, therefore no such system exists.

**18. (a)** For the given matrix to be non-singular

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$\Rightarrow 1-(a+c)\,\omega + a\,c\omega^2 \neq 0 \Rightarrow (1-a\omega)(1-c\omega) \neq 0$
$\Rightarrow a \neq \omega^2$ and $c \neq \omega^2$ where $\omega$ is complex cube root of unity.

As $a$, $b$ and $c$ are complex cube roots of unity
$\therefore$ $a$ and $c$ can take only one value i.e. $\omega$ while $b$ can take 2 values i.e. $\omega$ and $\omega^2$.
$\therefore$ Total number of distinct matrices $= 1 \times 1 \times 2 = 2$

**19. (d)** We have

$$|Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$= 2^2.2^3.2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2 a_{31} & 2^2 a_{32} & 2^2 a_{33} \end{vmatrix}$$

$$= 2^9.2.2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^{12} \times |P| = 2^{12} \times 2 = 2^{13}$$

**20. (d)** We have $P^T = 2P + I$
$\Rightarrow P = 2P^T + I \Rightarrow P = 2(2P + I) + I$
$\Rightarrow P = 4P + 3I \Rightarrow P + I = 0$
$\Rightarrow PX + X = 0 \Rightarrow PX = -X$

**21. (b)** $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = I + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} = I + A$

$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ $A^n = O, \forall\, n \geq 3$
Now $P^{50} = (I + A)^{50} = {}^{50}C_0 I^{50} + {}^{50}C_1 I^{49} A + {}^{50}C_2 I^{48} A^2 + O$
$= I + 50A + 25 \times 49 A^2.$
$\therefore$ $Q = P^{50} - I = 50A + 25 \times 49 A^2.$
$\Rightarrow$ $q_{21} = 50 \times 4 = 200 \Rightarrow q_{31} = 50 \times 16 + 25 \times 49 \times 16 = 20400$
$\Rightarrow$ $q_{32} = 50 \times 4 = 200$

$\therefore$ $\dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{20600}{200} = 103$

## D. MCQs with ONE or MORE THAN ONE Correct

**1. (b, e)** *ATQ* $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$

Operating $C_3 \to C_3 - C_1\alpha - C_2$, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha + b & b\alpha + c & -(a\alpha^2 + b\alpha + b\alpha + c) \end{vmatrix} = 0$$

$\Rightarrow$ $(a\alpha^2 + 2b\alpha + c) \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha + b & b\alpha + c & 1 \end{vmatrix} = 0$

$\Rightarrow$ $(ac - b^2)(a\alpha^2 + 2b\alpha + c) = 0$
$\Rightarrow$ either $ac - b^2 = 0$ or $a\alpha^2 + 2b\alpha + c = 0$
$\Rightarrow$ either $a, b, c$ are in G.P. or $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$
$\Rightarrow$ (b) and (e) are the correct answers.

**2. (d)** $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ (given)

$\Rightarrow$ $-3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$

[$\because$ $C_2$ and $C_3$ are identical]

$\Rightarrow$ $x + iy = 0 \Rightarrow x = 0, y = 0$

**3. (c)** [As a skew symmetric matrix of order 3 cannot be non singular, therefore the data given in the question is inconsistent.]
We have
$M^2N^2 (M^T N)^{-1} (MN^{-1})^T = M^2N^2N^{-1} (M^T)^{-1} (N^{-1})^T M^T$
$= M^2 N (M^T)^{-1} (N^{-1})^T M^T = -M^2NM^{-1} N^{-1}M$
($\because M^T = -M, N^T = -N$ and $(N^{-1})^T = (N^T)^{-1}$
$= -M (NM) (NM)^{-1} M$ ($\because MN = NM$)
$= -MM = -M^2$

**4. (a, d)**
We know for a third order matrix $P$, $|\text{Adj } P| = |P|^2$
Where $|\text{Adj } P| = 1(3-7) - 4(6-7) + 4(2-1) = 4$
$\therefore$ $|P|^2 = 4 \Rightarrow |P| = 2$ or $-2$

**5. (c, d)**
(a) $(N'MN)' = (MN)'N = N'M'N = N'MN$ or $-N'MN$
According as $M$ is symm. or skew symm. $\therefore$ correct
(b) $(MN - NM)' = (MN)' - (NM)' = N'M' - M'N'$
$= NM - MN = -(MN - NM)$
$\therefore$ It is skew symm. Statement $B$ is also correct.
(c) $(MN)' = N'M' = NM \neq MN$
$\therefore$ Statement $C$ is incorrect
(d) $(adj\, M)(adj\, N) = adj\,(MN)$ is incorrect.

**6. (b, c, d)**

For $n = 3$, $P = \begin{bmatrix} w^2 & w^3 & w^4 \\ w^3 & w^4 & w^5 \\ w^4 & w^5 & w^6 \end{bmatrix}$ and $P^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

It shows $P^2 = 0$ if n is a multiple of 3.
So for $P^2 \neq 0$, $n$ should not be a multiple of 3 i.e. $n$ can take values 55, 58, 56

**7.** **(c, d)** Let $M=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ where $a, b, c$ are integers.

$M$ is invertible if $\begin{vmatrix} a & b \\ b & c \end{vmatrix} \neq 0 \Rightarrow ac \neq b^2$

Then $\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a=b=c \Rightarrow ac=b^2$.

$\therefore$ (a) is not correct.

If $\begin{bmatrix} b & c \end{bmatrix}=\begin{bmatrix} a & b \end{bmatrix} \Rightarrow b=a=c \Rightarrow ac=b^2$

$\therefore$ (b) is not correct.

If $M=\begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$, then $|M|=ac \neq 0$

$\therefore$ $M$ is invertible.

(c) is correct

As $ac \neq (\text{integer})^2 \Rightarrow ac \neq b^2$

$\therefore$ (d) is correct.

**8.** **(a, b)** Given $MN=NM$, $M \neq N^2$ and $M^2=N^4$.

Then $M^2=N^4 \Rightarrow (M+N^2)(M-N^2)=0$

$\Rightarrow$ (i) $M+N^2=0$ and $M-N^2 \neq 0$

(ii) $|M+N^2|=0$ and $|M-N^2|=0$

In each case $|M+N^2|=0$

$\therefore$ $|M^2+MN^2|=|M|\,|M+N^2|=0$

$\therefore$ (a) is correct and (c) is not correct.

Also we know if $|A|=0$, then there can be many matrices $U$, such that $AU=0$

$\therefore$ $(M^2+MN^2)U=0$ will be true for many values of $U$.

Hence (b) is correct.

Again if $AX=0$ and $|A|=0$, then $X$ can be non-zero.

$\therefore$ (d) is not correct.

**9.** **(b, c)** $R_2-R_1,\quad R_3-R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 2\alpha+5 & 4\alpha+5 & 6\alpha+5 \end{vmatrix}=-648\alpha$$

$R_3-R_2$

$$2\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 1 & 1 & 1 \end{vmatrix}=-648\alpha$$

$C_2-C_1,\quad C_3-C_2$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(3\alpha+2) & \alpha(5\alpha+2) \\ 2\alpha+3 & 2\alpha & 2\alpha \\ 1 & 0 & 0 \end{vmatrix}=-324\alpha$$

$\Rightarrow 2\alpha^2(-2\alpha)=-324\alpha \Rightarrow \alpha^3-81\alpha=0 \Rightarrow \alpha=0, 9, -9$

**10.** **(c, d)**

$X'=-X,\ Y'=-Y,\ Z'=Z$

$(Y^3Z^4-Z^4Y^3)'=(Z^4)'(Y^3)'-(Y^3)'(Z^4)'$

$=(Z')^4(Y')^3-(Y')^3(Z')^4$

$=-Z^4Y^3+Y^3Z^4=Y^3Z^4-Z^4Y^3$

$\therefore$ Symmetric matrix.

Similarly $X^{44}+Y^{44}$ is symmetric matrix and $X^4Z^3-Z^3X^4$ and $X^{23}+Y^{23}$ are skew symmetric matrices.

**11.** **(b, c)** $PQ=kI \Rightarrow \dfrac{P.Q}{k}=I \Rightarrow P^{-1}=\dfrac{Q}{k}$

Also $|P|=12\alpha+20$

Comparing the third elements of 2nd row on both sides, we get

$-\left(\dfrac{3\alpha+4}{12\alpha+20}\right)=\dfrac{1}{k}\times\dfrac{-k}{8} \Rightarrow 24\alpha+32=12\alpha+20 \Rightarrow \alpha=-1$

$\therefore$ $|P|=8$

Also $PQ=kI \Rightarrow |P|\,|Q|=k^3$

$\Rightarrow 8\times\dfrac{k^2}{2}=k^3 \Rightarrow k=4 \Rightarrow |Q|=\dfrac{k^2}{2}=8$

(b) $4\alpha-k+8=4\times(-1)-4+8=0$

(c) Now det (P adj Q) = |P| adj Q|

$=|P|\,|Q|^2=8\times 8^2=2^9$

(d) $|Q \text{ adj } P|=|Q|\,|P|^2=2^9$

**12.** **(b, c, d)**

$ax+2y=\lambda$

$3x-2y=\mu$

For unique solution, $\dfrac{a}{3}\neq\dfrac{2}{-2} \Rightarrow a\neq -3$

$\therefore$ (b) is the correct option.

For infinite many solutions and $a=-3$

$\dfrac{-3}{3}=\dfrac{2}{-2}=\dfrac{\lambda}{\mu} \Rightarrow \dfrac{\lambda}{\mu}=-1$ or $\lambda+\mu=0$

$\therefore$ (c) is the correct option.

Also if $\lambda+\mu\neq 0$, then $\dfrac{-3}{3}=\dfrac{2}{-2}\neq\dfrac{\lambda}{\mu}$

$\Rightarrow$ system has no solution.

$\therefore$ (d) is the correct option.

## E. Subjective Problems

**1.** We should have,

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}=0$$

$\Rightarrow 1(-4k+6)-k(-12+4)+3(9-2k)=0$

$\Rightarrow -2k+33=0 \Rightarrow k=\dfrac{33}{2}$

Substituting $k=\dfrac{33}{2}$ and putting $x=b$, where $b\in Q$, we get the system as

$33y+6z=-2b$ ...(1)

$33y-4z=-6b$ ...(2)

$3y-4z=-2b$ ...(3)

$(1)-(2) \Rightarrow 10z=4b \Rightarrow z=\dfrac{2}{5}b$

$(1) \Rightarrow 33y=-2b-\dfrac{12b}{5}=-\dfrac{22b}{5} \Rightarrow y=\dfrac{-2b}{15}$

$\therefore$ The solution is $x=b,\ y=\dfrac{-2b}{15},\ z=\dfrac{2b}{5}$

**2.** The given det, on expanding along $R_1$, we get

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc-a^2)-b(b^2-ac)+c(ab-c^2)$$

$= 3abc - a^3 - b^3 - c^3 = -(a^3+b^3+c^3-3abc)$
$= -(a+b+c)[a^2+b^2+c^2-ab-bc-ca]$

$$= -\frac{1}{2}(a+b+c)[2a^2+2b^2+2c^2-2ab-2bc-2ca]$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

As $a, b, c > 0$
$\therefore \quad a+b+c>0$
Also $a \neq b \neq c$
$\therefore \quad (a-b)^2+(b-c)^2+(c-a)^2>0$
Hence the given determinant is – ve.

**3.**
$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA+B$$

$$\text{L.H.S.} = \begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$$

Operation $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

$$= \begin{vmatrix} x^2 & x+1 & x-2 \\ 0 & x-2 & x+1 \\ 0 & x-2 & x+1 \end{vmatrix} + \begin{vmatrix} x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

Operating $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} x & x+1 & x-2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} x & x & x \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$$

$$= x\begin{vmatrix} 1 & 1 & 1 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$$

$= xA + B =$ R.H. S
Hence Proved.

**4.** On L.H.S.$= D$, applying operations $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_2$ and using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$, we get

$$D = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

Operating $C_3 + C_2$ and using the same result, we get

$$D = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} = \text{RHS}$$

Hence proved

**5.** The system will have a non-trivial solution if

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

Expanding along $C_1$, we get
$\Rightarrow \quad (28-21)\sin 3\theta - (-7-7)\cos 2\theta + 2(-3-4) = 0$
$\Rightarrow \quad 7\sin 3\theta + 14\cos 2\theta - 14 = 0$
$\Rightarrow \quad \sin 3\theta + 2\cos 2\theta - 2 = 0$
$\Rightarrow \quad 3\sin - 4\sin^3\theta + 2(1-2\sin^2\theta) - 2 = 0$
$\Rightarrow \quad 4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$
$\Rightarrow \quad \sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$
$\sin\theta = 0$ or $\sin\theta = 1/2$ ($\sin\theta = -3/2$ not possible)
$\Rightarrow \quad \theta = n\pi$ or $\theta = n\pi + (-1)^n \pi/6, n \in Z$.

**6.** We have

$$\Delta a = \begin{vmatrix} (a-1) & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$\text{Then } \sum_{a=1}^{n} \Delta a = \begin{vmatrix} (1-1) & n & 6 \\ (1-1)^2 & 2n^2 & 4n-2 \\ (1-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$+ \begin{vmatrix} (2-1) & n & 6 \\ (2-1)^2 & 2n^2 & 4n-2 \\ (2-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix} + \ldots\ldots$$

$$+ \begin{vmatrix} (n-1) & n & 6 \\ (n-1)^2 & 2n^2 & 4n-2 \\ (n-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \begin{vmatrix} 1+2+3+\ldots+(n-1) & n & 6 \\ 1^2+2^2+3^2+\ldots\ldots+(n-1)^2 & 2n^2 & 4n-2 \\ 1^3+2^3+3^3+\ldots+(n-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4n-2 \\ \left(\frac{n(n-1)}{2}\right)^2 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \frac{n^2(n-1)}{12} \begin{vmatrix} 6 & 1 & 6 \\ 2(2n-1) & 2n & 2(2n-1) \\ 3n(n-1) & 3n^2 & 3n(n-1) \end{vmatrix}$$

(Taking $\frac{n(n-1)}{12}$ common from $C_1$ and $n$ from $C_2$)

$= 0$ (as $C_1$ and $C_3$ are identical)

Thus, $\sum_{a=1}^{n} \Delta a = 0 \Rightarrow \sum_{a=1}^{n} \Delta a = c$ (a constant) where $c = 0$

7. Given that $A, B, C$ are integers between 0 and 9 and the three digit numbers $A28$, $3B9$ and $62C$ are divisible by a fixed integer $k$.

Now, $D = \begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$

On operating $R_2 \to R_2 + 10R_3 + 100R_1$, we get

$$= \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ kn_1 & kn_2 & kn_3 \\ 2 & B & 2 \end{vmatrix}$$

[As per question $A28$, $3B9$ and $62C$ are divisible by $k$, therefore,

$A28 = kn_1$
$3B9 = kn_2$
$62C = kn_3$]

$$= k \begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix} = k \times \text{some integral value.}$$

$\Rightarrow D$ is divisible by $k$.

8. Consider $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Operating $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ we get

$$\begin{vmatrix} p-a & -(q-b) & c \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0$$

Taking $(p-q)$, $(q-b)$ and $(r-c)$ common from $C_1$, $C_2$ and $C_3$ resp, we get

$$\Rightarrow (p-a)(q-b)(r-c) \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

Expanding along $R_1$

$$\Rightarrow (p-a)(q-b)(r-c)\left[1\left(\frac{r}{r-c} + \frac{b}{q-b}\right) + \frac{a}{p-a}\right] = 0$$

As given that $p \neq a, q \neq b, r \neq c$

$$\therefore \frac{r}{r-c} + \frac{b}{q-b} + \frac{a}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + \frac{q-(q-b)}{q-b} + \frac{p-(p-a)}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + \frac{q}{q-b} - 1 + \frac{p}{p-a} - 1 = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

9. $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$

$$= n!(n+1)!(n+2)! \begin{vmatrix} 1 & n+1 & (n+2)(n+1) \\ 1 & n+2 & (n+3)(n+2) \\ 1 & n+3 & (n+4)(n+3) \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$, we get

$$D = (n!)^3 (n+1)^2 (n+2) \begin{vmatrix} 1 & n+1 & n^2+3n+2 \\ 0 & 1 & 2n+4 \\ 0 & 1 & 2n+6 \end{vmatrix}$$

Operating $R_3 \to R_3 - R_2$

$$D = (n!)^3 (n+1)^2 (n+2) \begin{vmatrix} 1 & n+1 & n^2+3n+2 \\ 0 & 1 & 2n+4 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= (n!)^3 (n+1)^2 (n+2)\, 1\, [2]$$

$$\Rightarrow \frac{D}{(n!)^3} = 2\,(n+1)^2(n+2)$$

$$\Rightarrow \frac{D}{(n!)^3} - 4 = 2(n^3 + 4n^2 + 5n + 2) - 4$$

$$= 2(n^3 + 4n^2 + 5n) = 2n(n^2 + 4n + 5)$$

$\Rightarrow \frac{D}{(n!)^3} - 4$ is divisible by $n$.

10. Given that $\lambda, \alpha \in R$ and system of linear equations

$\lambda x + (\sin\alpha) y + (\cos\alpha) z = 0$
$x + (\cos\alpha) y + (\sin\alpha) z = 0$
$-x (\sin\alpha) y - (\cos\alpha) z = 0$

has a non trivial solution, therefore $D = 0$

$$\Rightarrow \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - \sin\alpha(-\cos\alpha + \sin\alpha) + \cos\alpha(\sin\alpha + \cos\alpha) = 0$$

$\Rightarrow -\lambda + \sin\alpha\cos\alpha - \sin^2\alpha + \sin\alpha\cos\alpha + \cos^2\alpha = 0$
$\Rightarrow \lambda = \cos^2\alpha - \sin^2\alpha + 2\sin\alpha\cos\alpha$
$\Rightarrow \lambda = \cos 2\alpha + \sin 2\alpha$
For $\lambda = 1$, $\cos 2\alpha + \sin 2\alpha = 1$

$$\frac{1}{\sqrt{2}}\cos 2\alpha + \frac{1}{\sqrt{2}}\sin 2\alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos 2\alpha \cos \pi/4 + \sin 2\alpha \sin \pi/4 = \frac{1}{\sqrt{2}}$$

$\Rightarrow \cos(2\alpha - \pi/4) = \cos\pi/4 \Rightarrow 2\alpha - \pi/4 = 2n\pi \pm \pi/4$
$\Rightarrow 2\alpha = 2\pi r + \pi/4 + \pi/4;\ 2n\pi - \pi/4 + \pi/4$
$\Rightarrow \alpha = n\pi + \pi/4$ or $n\pi$

**11.** L.H.S.

$$= \begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos(A-Q) & \cos(A-R) \\ \cos B\cos P + \sin B\sin P & \cos(B-Q) & \cos(B-R) \\ \cos C\cos P + \sin C\sin P & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$= \cos P \begin{vmatrix} \cos A & \cos(A-Q) & \cos(A-R) \\ \cos B & \cos(B-Q) & \cos(B-R) \\ \cos C & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$+ \sin P \begin{vmatrix} \sin A & \cos(A-Q) & \cos(A-R) \\ \sin B & \cos(B-Q) & \cos(B-R) \\ \sin C & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

Operating; $C_2 \to C_2 - C_1(\cos Q)$; $C_3 \to C_3 - C_1(\cos R)$ on first determinant and $C_2 \to C_2 - (\sin Q)C_1$ and $C_3 \to C_3 - (\sin R)C_1$ on second determinant, we get

$$= \cos P \begin{vmatrix} \cos A & \sin A\sin Q & \sin A\sin R \\ \cos B & \sin B\sin Q & \sin B\sin R \\ \cos C & \sin C\sin Q & \sin C\sin R \end{vmatrix}$$

$$+ \sin P \begin{vmatrix} \sin A & \cos A\cos Q & \cos A\cos R \\ \sin B & \cos B\cos Q & \cos B\cos R \\ \sin C & \cos C\cos Q & \cos C\cos R \end{vmatrix}$$

$$= \cos P\sin Q\sin R \begin{vmatrix} \cos A & \sin A & \sin A \\ \cos B & \sin B & \sin B \\ \cos C & \sin C & \sin C \end{vmatrix}$$

$$+ \sin P\cos Q\cos R \begin{vmatrix} \sin A & \cos A & \cos A \\ \sin B & \cos B & \cos B \\ \sin C & \cos C & \cos C \end{vmatrix}$$

$= 0 + 0$ [Both determinants become zero as $C_2 \equiv C_3$]
$= 0 =$ R.H.S.

**12.** Let us denote the given determinant by $\Delta$. Taking $\frac{1}{a(a+d)(a+2d)}$ as common from $R_1$, $\frac{1}{(a+d)(a+2d)(a+3d)}$ from $R_2$ and $\frac{1}{(a+2d)(a+3d)(a+4d)}$ from $R_3$, we get

$$\Delta = \frac{1}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}\Delta_1$$

where

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(a+3d) & a+3d & a+d \\ (a+3d)(a+4d) & a+4d & a+2d \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$, we get

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(2d) & d & d \\ (a+3d)(2d) & d & d \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(2d) & d & d \\ 2d^2 & 0 & 0 \end{vmatrix}$$

Expanding along $R_3$, we get

$$\Delta_1 = (2d^2)\begin{vmatrix} a+2d & a \\ d & d \end{vmatrix} = (2d)^2(d)(a+2d-a) = 4d^4$$

$$\text{Thus, } \Delta = \frac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$$

**13.** $R_2 \to R_2 + R_3$,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ 2\sin\theta\cos\frac{2\pi}{2} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3} \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

$$= \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

**14.** Given that $A^T A = I$
$\Rightarrow |A^T A| = |A^T||A| = |A||A| = 1$ $\quad [\because |I| = I]$
$\Rightarrow |A|^2 = 1$ ....(1)

$$\text{From given matrix } A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc \quad ....(2)$$

$\therefore (a^3 + b^3 + c^3 - 3abc)^2 = 1$ (From (1) and (2)]
$\Rightarrow a^3 + b^3 + c^3 - 3abc = 1$ or $-1$
But for $a^3, b^3, c^3$ using $AM \geq GM$

We get $\frac{a^3+b^3+c^3}{3} \geq \sqrt[3]{a^3b^3c^3} \Rightarrow a^3 + b^3 + c^3 - 3abc \geq 0$

$\therefore$ We must have $a^3 + b^3 + c^3 - 3abc = 1$
$\Rightarrow a^3 + b^3 + c^3 = 1 + 3 \times 1 = 4$ [Using $abc = 1$]

**15.** We are given that $MM^T = I$ where $M$ is a square matrix of order 3 and det. $M = 1$.

Consider det $(M-I) = \det(M - MM^T)$ [Given $MM^T = I$]
$= \det[M(I - M^T)]$
$= (\det M)(\det(I - M^T))$
[$\because |AB| = |A||B|$]
$= -(\det M)(\det(M^T - I))$
$= -1[\det(M^T - I)]$ [$\because \det(M) = 1$]
$= -\det(M - I)$
[$\because \det(M^T - I) = \det[(M-I)^T] = \det(M - I)$]
$\Rightarrow 2\det(M-I) = 0 \Rightarrow \det(M-I) = 0$
Hence Proved

**16.** Given that,

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

and $AX = U$ has infinite many solutions.
$\Rightarrow |A| = 0 = |A_1| = |A_2| = |A_3|$
Now, $|A| = 0$

$$\Rightarrow \begin{vmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{vmatrix} = a(bc - bd) - 1(c - d) = 0$$

$\Rightarrow (ab - 1)(c - d) = 0$
$\Rightarrow ab = 1$ or $c = d$ ..........(1)

$$\text{And } |A_1| = \begin{vmatrix} f & 1 & 0 \\ g & b & d \\ h & b & c \end{vmatrix} = 0$$

$\Rightarrow f(bc - bd) - 1(gc - hd) = 0$
$\Rightarrow fb(c - d) = gc - hd$ ...........(2)

$$|A_2| = \begin{vmatrix} a & 1 & f \\ 1 & b & g \\ 1 & b & h \end{vmatrix} = 0$$

$\Rightarrow a(gc - hd) - f(c - d) = 0 \Rightarrow a(gc - hd) = f(c - d)$

$$|A_3| = \begin{vmatrix} a & 1 & f \\ 1 & b & g \\ 1 & b & h \end{vmatrix} = 0$$

$\Rightarrow a(bh - bg) - 1(h - g) + f(b - b) = 0$
$\Rightarrow ab(h - g) - (h - g) = 0$
$\Rightarrow ab = 1$ or $h = g$ .........(3)
Taking $c = d \Rightarrow h = g$ and $ab \neq 1$ (from (1), (2) and (3))
Now taking $BX = V$

$$\text{where } B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Then } |B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0$$

[$\because$ In vew of $c = d$ and $g = h$, $C_2$ and $C_3$ are identical]
$\Rightarrow BX = V$ has no unique solution

$$\text{And } |B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0 \quad (\because c = d, g = h)$$

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2cf = a^2df \quad (\because c = d)$$

$$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a^2df$$

$\Rightarrow$ If $adf \neq 0$ then $|B_2| = |B_3| \neq 0$
Hence no solution exist.

## F. Match the Following

**1.** The given lines are
$L_1 : x + 3y - 5 = 0$
$L_2 : 3x - ky - 1 = 0$
$L_3 : 5x + 2y - 12 = 0$

(A) Three lines $L_1, L_2, L_3$ are concurrent if

$$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -k & 1 \\ 5 & 2 & 12 \end{vmatrix} = 0 \Rightarrow 13k - 65 = 0 \Rightarrow k = 5$$

$\therefore$ (A) $\rightarrow$ (s)

(B) For $L_1 \parallel L_2 \Rightarrow \frac{1}{3} = \frac{-3}{k} \Rightarrow k = -9$

and $L_2 \parallel L_3 \Rightarrow \frac{3}{5} = \frac{-k}{2} \Rightarrow k = -\frac{6}{5}$

$\therefore$ (B) $\rightarrow$ (p), (q)

(C) Three lines $L_1, L_2, L_3$ will form a triangle if no two of them are parallel and no three are concurrent
$\therefore k \neq 5, -9, -6/5 \quad \therefore$ (C) $\rightarrow$ r

(D) $L_1, L_2, L_3$ do not form a triangle if either any two of these are parallel or the three are concurrent i.e.
$k = 5, -9, -6/5$
$\therefore$ (D) $\rightarrow$ (p), (q), (s)

**2.** (A) Let $y = \frac{x^2 + 2x + 4}{x + 2} \Rightarrow \frac{dy}{dx} = \frac{x^2 + 4x}{(x+2)^2} = 0$

$\Rightarrow x = 0, -4$

$$\frac{d^2y}{dx^2} = \frac{8}{(x+2)^3}$$

At $x = 0$, $\frac{d^2y}{dx^2}$ is true

$\therefore$ y is min when x = 0, $\therefore$ ymin = 2

(B) As A is symmetric and B is skew symmetric matrix, we should have

$A^t = A$ and $B^t = -B$ ...(1)

Also given that

$(A+B)(A-B) = (A-B)(A+B)$

$\Rightarrow A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow 2BA = 2AB$ or $AB = BA$ ...(2)

Now given that

$(AB)^t = (-1)^k AB$

$\Rightarrow (BA)^t = (-1)^k AB$ (using equation (2))

$\Rightarrow A^t B^t = (-1)^k AB$

$\Rightarrow -AB = (-1)^k AB$ [using equation(1)]

$\Rightarrow k$ should be an odd number

$\therefore$ (B) $\rightarrow$ (q), (s)

(C) Given that $a = \log_3 \log_3 2$

$\Rightarrow \log_3 2 = 3^a \Rightarrow \frac{1}{\log_2 3} = 3^a$ or $\log_2 3 = 3^{-a}$

$\Rightarrow \quad 3 = 2^{(3^{-a})}$ ...(1)

Now $1 < 2^{(-k+3^{-a})} < 2 \Rightarrow 1 < 2^{-k}.2^{3^{-a}} < 2$

$\Rightarrow 1 < 2^{-k}.3 < 2$ (using eq (1))

$\Rightarrow \frac{1}{3} < 2^{-k} < \frac{2}{3} \Rightarrow \frac{3}{2} < 2^k < 3 \Rightarrow k = 1$

$\therefore k$ is less than 2 and 3

$\therefore$ (C) $\rightarrow$ (r), (s).

(D) Given that $\sin\theta = \cos\phi \Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi$

$\Rightarrow \frac{\pi}{2} - \theta = 2n\pi \pm \phi,\ n \in Z \Rightarrow \theta \pm \phi - \frac{\pi}{2} = -2n\pi$

$\Rightarrow \frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n$

$\therefore$ Here possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are 0 and 2 for $n = 0, -1$.

$\therefore$ D $\rightarrow$ (p), (r).

## G. Comprehension Based Questions

1. (d) Let $U_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ then $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow a = 1, b = -2, c = 1$

$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ Similarly, $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

$\therefore U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \Rightarrow |U| = 3$

2. (b) $U^{-1} = \frac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

$\Rightarrow$ Sum of elements of $U^{-1} = \frac{1}{3}(0) = 0$

3. (a) $[3\ 2\ 0]\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = [3\ 2\ 0]\begin{bmatrix} 7 \\ -8 \\ -5 \end{bmatrix} = 5$

4. (a) Each element of set $A$ is $3 \times 3$ symmetric matrix with five of its entries as 1 and four of its entries as 0, we can keep in diagonal either 2 zero and one 1 or no zero and three 1 so that the left over zeros and one's are even in number.

Hence taking 2 zeros and one 1 in diagonal the possible cases are $\frac{3!}{2!} \times \frac{3!}{2!} = 9$

and taking 3 ones in diagonal the possible cases are

$1 \times \frac{3!}{2!} = 3$

$\therefore$ Total elements $A$ can have $= 9 + 3 = 12$

5. (b) The given system will have unique solution if $|A| \neq 0$ which is so for the matrices.

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$;

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$;

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

which are 6 in number.

6. (b) For the given system to be inconsistent $|A| = 0$. The matrices for which $|A| = 0$ are

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (i), $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (ii), $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (iii)

and $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (iv), $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (v), $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (vi)

On solving $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$

We find for $A = $ (i)

By Cramer's rule $D_1 = 0 = D_2 = D_3$

$\therefore$ infinite many solution

For $A = $ (ii)

By Cramer's rule $D_1 \neq 0$

$\Rightarrow$ no solution i.e. inconsistent.

Similarly we find the system as inconsistent in cases (iii), (v) and (vi).

Hence for four cases system is inconsistent.

**7.** **(d)** If A is symmetrie then b = c

$\Rightarrow |A| = a^2 - b^2 = (a+b)(a-b)$

Which is divisible by $p$ if $(a + b)$ is divisible by $p$ or $(a - b)$ is divisible by $p$.

Now $(a + b)$ is divisible by $p$ if $(a, b)$ can take values $(1, p-1), (2, p-2), (3, p-3), ...(p-1, 1)$

$\therefore (p-1)$ ways.

Also $(a - b)$ is divisible by p only when $a - b = 0$

i.e. $a = b$, then $(a, b)$ can take values (0,0) (1,1), (2,2)..... (pÝ, p–2)

$\therefore p$ ways.

If $A$ is skew symmetric, then $a = 0$ and $b = -c$ or $b + c = 0$ which gives $|A| = 0$ when $b^2 \Rightarrow b = 0, c = 0$

But this possibility is already included when $A$ is symmetric and $(a, b) = (0, 0)$.

Again if $A$ is both symmetric and skew symmetric, then clearly $A$ is null matrix which case is already included.

Hence total number of ways $= p + (p - 1) = 2p - 1$

**8.** **(c)** Trace $A = a + a = 2a$ is not divisible by $p$

$\Rightarrow a$ is not divisible by $p \Rightarrow a \neq 0$

But $|A|$ is divisible by $p \Rightarrow a^2 - bc$ is divisible by $p$

It will be so if on dividing $a^2$ by p suppose we get $m\frac{l}{p}$

then on dividing bc by p we should get $n\frac{l}{p}$ for some integeral values of m, n and $l$.

i.e. the remainder should be same in each case, so that

$$\frac{a^2 - bc}{p} = \left(m + \frac{l}{p}\right) - \left(n + \frac{l}{p}\right) = (m - n) = \text{an integer}$$

For this to happen a can take any value from 1 to $p$–1, also if $b$ takes any value from 1 to $p$–1 then $c$ should take only that value corresponding to which the remainder is same.

$\therefore$ No. of ways $= (p-1) \times (p-1) \times 1 = (p-1)^2$.

**9.** **(d)** Total number of matrices

= total number of ways $a, b, c$ can be selected

$= p \times p \times p = p^3$.

Number of ways when det (A) is divisible by p and trace (A) $\neq$ 0 are equal to number of ways det (A) is divisible by p and trace (A) is not divisible by $p = (p-1)^2$

Also number of ways when det (A) is divisible by $p$ and trace A = 0 are the ways when bc is multiple of $p$

$\Rightarrow b = 0$ or $c = 0$

for $b = 0$, $c$ can take values $0, 1, 2, ......., p-1$

For $c = 0$, $b$ can take values $0, 1, 2, ..........., p-1$

Here $(b, c) = (0, 0)$ is coming twice.

$\therefore$ Total ways of selecting $b$ and $c = p + p - 1$
$= 2p - 1$

$\therefore$ Total number of ways when det (A) is divisible by $p = (p-1)^2 + 2p - 1 = p^2$

Hence the number of ways when det (A) is not divisible by $p = p^3 - p^2$.

**10.** **(d)** From equation (E), we get

$a + 8b + 7c = 0$
$9a + 2b + 3c = 0$
$a + b + c = 0$

Here $\begin{vmatrix}1 & 8 & 7\\9 & 2 & 3\\1 & 1 & 1\end{vmatrix} = 0$,

Therefore system has infinite many solutions.

Solving these, we get $b = 6a$ and $c = -7a$

Now $(a, b, c)$ lies on $2x + y + z = 1 \Rightarrow b = 6, c = -7$

$\therefore\ 2a + 6a - 7a = 1 \Rightarrow a = 1$

$\therefore\ 7a + b + c = 7 + 6 - 7 = 6 \Rightarrow b = 6, c = -7$

**11.** **(a)** If $a = 2$ then $b = 12, c = -14$

$\therefore \frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}}$

$= 3\omega + 1 + 3\omega^2 = 1 - 3 = -2$

**12.** **(b)** If $b = 6$ then $a = 1, c = -7$

$\therefore$ Equation becomes $x^2 + 6x - 7 = 0$ or $(x + 7)(x - 1) = 0$ whose roots are 1 and –7.

Let $\alpha = 1$ and $\beta = -7$

$$\therefore \sum_{n=0}^{\infty}\left(\frac{1}{1} - \frac{1}{7}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = \frac{1}{1 - \frac{6}{7}} = 7$$

## H. Assertion & Reason Type Questions

**1.** **(a)** The given equations are

$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

Here $D = \begin{vmatrix}1 & -2 & 3\\-1 & 1 & -2\\1 & -3 & 4\end{vmatrix} = 0$

and $D_2 = \begin{vmatrix}1 & -1 & 3\\-1 & k & -2\\1 & 1 & 4\end{vmatrix} = k - 3 \neq 0$ if $k \neq 3$

$\therefore$ If $k \neq 3$, the system has no solutions.

Hence statement-1 is true and statement-2 is a correct explanation for statement - 1.

## I. Integer Value Correct Type

**1.** **(0)** We have $\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \frac{-1+i\sqrt{3}}{2}$

$\therefore 1+\omega+\omega^2 = 0$ and $\omega^3 = 1$

Then $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$C_1 \leftrightarrow C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} z+1+\omega+\omega^2 & \omega & \omega^2 \\ z+1+\omega+\omega^2 & z+\omega^2 & 1 \\ z+1+\omega+\omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$\Rightarrow z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$

$\Rightarrow z\left[1\left(z^2+z\omega+z\omega^2+\omega^3-1\right)-\omega\left(z+\omega-1\right)+\omega^2\left(1-z-\omega^2\right)=0\right]$

$\Rightarrow z\left[z^2+z\omega+z\omega^2-z\omega-\omega^2+\omega+\omega^2-z\omega^2-\omega^4\right]=0$

$\Rightarrow z\left[z^2\right]=0 \Rightarrow z^3=0 \Rightarrow z=0$

$\therefore z = 0$ is the only solution.

**2.** **(4)** $|A| = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}, C_2 \to C_2 - C_3$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}, R_2 \to R_2 - R_3$

$= (1+2k)(8k-4k+4k^2+1) = (2k+1)^3$

Also $|B| = 0$ as $B$ is skew symmetric of odd order.

$\therefore |\text{Adj } A| + |\text{Adj } B| = |A|^2 + |B|^2 = 10^6$

$\Rightarrow (2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10 \Rightarrow k = 4.5$

$\therefore [k] = 4$

**3.** **(9)**

Let $M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

then $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{matrix} b_1 = -1 \\ b_2 = 2 \\ b_3 = 3 \end{matrix}$

$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & c_3 & c_3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \begin{matrix} a_1 - b_1 = 1 \\ a_2 - b_2 = 1 \\ a_3 - b_3 = -1 \end{matrix}$

$\Rightarrow a_1 = 0, a_2 = 3, a_3 = 2$

and $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow a_3 + b_3 + c_3 = 12$
$\Rightarrow c_3 = 7$

$\therefore$ Sum of diagonal elements $= a_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

**4.** **(2)** $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$

$\Rightarrow x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$

Operating $C_2 - C_1, C_3 - C_1$ for both the determinants, we get

$\Rightarrow x^3\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$

$\Rightarrow x^3(-4+6) + x^6(48-36) = 10$
$\Rightarrow 2x^3 + 12x^6 = 10 \Rightarrow 6x^6 + x^3 - 5 = 0$

$\Rightarrow (6x^3 - 5)(x^3 + 1) = 0 \Rightarrow x = \left(\frac{5}{6}\right)^{\frac{1}{3}}, -1$

**5.** **(1)** $z = \frac{-1+i\sqrt{3}}{2} \Rightarrow z^3 = 1$ and $1 + z + z^2 = 0$

$P^2 = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$

$= \begin{bmatrix} z^{2r} + z^{4s} & z^{2s}\left((-z)^r + z^r\right) \\ z^{2s}\left((-z)^r + z^r\right) & z^{4s} + z^{2r} \end{bmatrix}$

For $P^2 = -I$ we should have
$z^{2r} + z^{4s} = -1$ and $z^{2s}((-z)^r + z^r) = 0$
$\Rightarrow z^{2r} + z^s + 1 = 0$ and $(-z)^r + z^r = 0$
$\Rightarrow$ r is odd and s = r but not a multiple of 3.
Which is possible when s = r = 1
$\therefore$ only one pair is there.

# Section-B JEE Main/ AIEEE

1. **(c)** We have $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

By $R_3 \to R_3-(xR_1+R_2)$;

$$=\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+C) \end{vmatrix}$$

$=(ax^2+2bx+c)(b^2-ac)=(+)(-)=-\text{ve}.$

2. **(d)** For homogeneous system of equations to have non zero solution, $\Delta=0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix}=0 \quad C_2 \to C_2-2C_3$$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix}=0 \quad R_3 \to R_3-R_2, R_2 \to R_2-R_1$$

$$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix}=0$$

$b(c-b)-(b-a)(2c-b)=0$

On simplification, $\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$

$\therefore a,b,c$ are in Harmonic Progression.

3. **(b)** $\Delta=\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$$=1\left(\omega^{3n}-1\right)-\omega^n\left(\omega^{2n}-\omega^{2n}\right)+\omega^{2n}\left(\omega^n-\omega^{4n}\right)$$

$=\omega^{3n}-1-0+\omega^{3n}-\omega^{6n}$

$=1-1+1-1=0\left[\because \omega^{3n}=1\right]$

4. **(c)** $A^2=\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}=\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$$=\begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$$

$\alpha=a^2+b^2;\ \beta=2ab$

5. **(a)** $A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A|=-1 \neq 0$

$\therefore A^{-1}$ exists, further $(-1)I=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also $A^2=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

6. **(a)** Given that $10B=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$\Rightarrow B=\frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Also since, $B=A^{-1} \Rightarrow AB=I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{5-\alpha}{10}=0 \Rightarrow \alpha=5$$

7. **(d)** Let $r$ be the common ratio, then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$=\begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$=\begin{vmatrix} \log a_1+(n-1)\log r & \log a_1+n\log r & \log a_1+(n+1)\log r \\ \log a_1+(n+2)\log r & \log a_1+(n+3)\log r & \log a_1+(n+4)\log r \\ \log a_1+(n+5)\log r & \log a_1+(n+6)\log r & \log a_2+(n+7)\log r \end{vmatrix}$$

$$=0 \quad \left[\text{Apply } c_2 \to c_2-\frac{1}{2}c_1-\frac{1}{2}c_3\right]$$

**8.** **(d)** Given $A^2 - A + I = 0$

$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$

(Multiplying $A^{-1}$ on both sides)

$\Rightarrow A - 1 + A^{-1} = 0$ or $A^{-1} = 1 - A$.

**9.** **(a)** $\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1;$

$x + y + z\alpha = \alpha - 1$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$

$= \alpha\,(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$

For infinite solutions, $\Delta = 0$

$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 1 - 1] = 0$

$\Rightarrow (\alpha - 1)[\alpha^2 + \alpha - 2] = 0 \Rightarrow \alpha = -2, 1;$

But $\alpha \neq 1$. $\therefore \alpha = -2$

**10.** **(d)** Applying, $C_1 \to C_1 + C_2 + C_3$ we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2x) \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2x) \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

[As given that $a^2 + b^2 + c^2 = -2$]

$\therefore a^2 + b^2 + c^2 + 2 = 0$

Applying $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\therefore f(x) = \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$f(x) = (x-1)^2$ Hence degree = 2.

**11.** **(b)** $\because a_1, a_2, a_3, .......$ are in G.P.

$\therefore$ Using $a_n = ar^{n-1}$, we get the given determinant, as

$$\begin{vmatrix} \log ar^{n-1} & \log ar^n & \log ar^{n+1} \\ \log ar^{n+2} & \log ar^{n+3} & \log ar^{n+4} \\ \log ar^{n+5} & \log ar^{n+6} & \log ar^{n+7} \end{vmatrix}$$

Operating $C_3 - C_2$ and $C_2 - C_1$ and using

$\log m - \log n = \log \frac{m}{n}$ we get

$$= \begin{vmatrix} \log ar^{n-1} & \log r & \log r \\ \log ar^{n+2} & \log r & \log r \\ \log ar^{n+5} & \log r & \log r \end{vmatrix}$$

= 0 (two columns being identical)

**12.** **(b)** $A^2 - B^2 = (A - B)(A + B)$

$A^2 - B^2 = A^2 + AB - BA - B^2 \quad \Rightarrow AB = BA$

**13.** **(d)** $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

Hence, $AB = BA$ only when $a = b$

$\therefore$ There can be infinitely many $B$'s for which $AB = BA$

**14.** **(d)** Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$ and $R \to R_3 - R_1$

$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, $D$ is divisible by both $x$ and $y$

**15.** **(a)** $|A^2| = 25 \Rightarrow |A|^2 = 25 \Rightarrow (25\alpha)^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$

**16.** **(d)** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = I$

$\Rightarrow a^2 + bc = 1 \quad ab + bd = 0$

$ac + cd = 0 \quad bc + d^2 = 1$

From these four relations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $b(a + d) = 0 = c(a + d) \Rightarrow a = -d$

We can take $a = 1, b = 0, c = 0, d = -1$ as one possible set of values, then $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Clearly $A \neq I$ and $A \neq -I$ and $\det A = -1$

$\therefore$ Statement 1 is true.

Also if $A \neq I$ then $tr(A) = 0$

$\therefore$ Statement 2 is false.

**17.** **(d)** The given equations are

$-x + cy + bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

$\because x, y, z$ are not all zero

$\therefore$ The above system should not have unique (zero) solution

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow -1(1-a^2) - c(-c-ab) + b(ac+b) = 0$
$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

**18. (c)** $\because$ All entries of square matrix $A$ are integers, therefore all cofactors should also be integers.
If det $A = \pm 1$ then $A^{-1}$ exists. Also all entries of $A^{-1}$ are integers.

**19. (d)** We know that $|adj\,(adj\,A)| = |\text{adj A}|^{2-1}$
$= |A|^{2-1} = |A|$
$\therefore$ Both the statements are true and statement -2 is a correct explantion for statement-1 .

**20. (b)** $$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$$

(Taking transpose of second determinant)

$C_1 \Leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$$

$C_2 \Leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$C_2 - C_1,\ C_3 - C_1$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$R_1 + R_3$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $[1+(-1)^{n+2}](a+c)(2b+1+2b-1) = 0$
$\Rightarrow$ $4b(a+c)[1+(-1)^{n+2}] = 0$
$\Rightarrow$ $1+(-1)^{n+2} = 0$ as $b(a+c) \neq 0$
$\Rightarrow$ $n$ should be an odd integer.

**21. (c)** $\begin{bmatrix} 1 & ... & ... \\ ... & 1 & ... \\ ... & ... & 1 \end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

Similarly, $\begin{bmatrix} ... & ... & 1 \\ ... & 1 & ... \\ 1 & ... & ... \end{bmatrix}$ are 6 non-singular matrices.

So, required cases are more than 7, non-singular $3 \times 3$ matrices.

**22. (b)** Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d $\neq 0$

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$
$ab + bd = ac + cd = 0$
$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0 \Rightarrow \text{Tr(A)} = 0$
$|A| = ad - bc = -a^2 - bc = -1$

**23. (c)** $$D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0 \qquad D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$$

$\Rightarrow$ Given system, does not have any solution.
$\Rightarrow$ No solution

**24. (a)** $$\Delta = 0 \Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 4(4-2) - k(k-2) + 2(2k-8) = 0$
$\Rightarrow 8 - k^2 + 2k + 4k - 16 = 0 \quad k^2 - 6k + 8 = 0$
$\Rightarrow (k-4)(k-2) = 0,\ k = 4, 2$

**25. (a)** $\therefore$ A′ = A, B′ = B
Now (A(BA))′ = (BA)′A′
= (A′B′)A′ = (AB)A = A(BA)
Similarly ((AB)A)′ = (AB)A
So, A(BA) and (AB)A are symmetric matrices.
Again (AB)′ = B′A′ = BA
Now if BA = AB, then AB is symmetric matrix.

**26. (d)** Let $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Then, $Au_1 + Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \quad A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad ...(1)$$

Also, $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix} \Rightarrow |A|=1(1)-0(2)+0(4-3)=1$

We know,

$A^{-1}=\frac{1}{|A|}adjA \Rightarrow A^{-1}=adj(A) \quad (\because |A|=1)$

Now, from equation (1), we have

$u_1+u_2=A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix} \Rightarrow u_1+u_2=A^{-1}$

$=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

**27. (c)** Given $P^3=Q^3$ ...(1)
and $P^2Q=Q^2P$ ...(2)
Subtracting (1) and (2), we get
$P^3-P^2Q=Q^3-Q^2P$
$\Rightarrow P^2(P-Q)+Q^2(P-Q)=0$
$\Rightarrow (P^2+Q^2)(P-Q)=0 \Rightarrow |P^2+Q^2|=0$ as $P\neq Q$

**28. (b)** $|P|=1(12-12)-\alpha(4-6)+3(4-6)=2\alpha-6$
Now, adj A = P $\Rightarrow |\text{adj }A|=|P|$
$\Rightarrow |A|^2=|P| \Rightarrow |P|=16$
$\Rightarrow 2\alpha-6=16 \Rightarrow \alpha=11$

**29. (a)** Consider

$\begin{vmatrix}3&1+f(1)&1+f(2)\\1+f(1)&1+f(2)&1+f(3)\\1+f(2)&1+f(3)&1+f(4)\end{vmatrix}$

$=\begin{vmatrix}1+1+1&1+\alpha+\beta&1+\alpha^2+\beta^2\\1+\alpha+\beta&1+\alpha^2+\beta^2&1+\alpha^3+\beta^3\\1+\alpha^2+\beta^2&1+\alpha^3+\beta^3&1+\alpha^4+\beta^4\end{vmatrix}$

$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}\times\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}$

$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}^2=[(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

So, $\boxed{k=1}$

**30. (d)** $BB'=B(A^{-1}A')'=B(A')'(A^{-1})'=BA(A^{-1})'$

$=(A^{-1}A')(A(A^{-1})')$
$=A^{-1}A.A'.(A^{-1})'$ {as $AA'=A'A$}
$=I(A^{-1}A)'$
$=I.I=I^2=I$

**31. (a)** $\left.\begin{aligned}2x_1-2x_2+x_3&=\lambda x_1\\2x_1-3x_2+2x_3&=\lambda x_2\\-x_1+2x_2&=\lambda x_3\end{aligned}\right\}$

$\Rightarrow (2-\lambda)x_1-2x_2+x_3=0$
$2x_1-(3+\lambda)x_2+2x_3=0$
$-x_1+2x_2-\lambda x_3=0$
For non-trivial solution, $\Delta=0$

i.e. $\begin{vmatrix}2-\lambda&-2&1\\2&-(3+\lambda)&2\\-1&2&-\lambda\end{vmatrix}=0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda)-4]+2[-2\lambda+2]+1[4-(3+\lambda)]=0$
$\Rightarrow \lambda^3+\lambda^2-5\lambda+3=0 \Rightarrow \lambda=1,1,3$
Hence, $\lambda$ has 2 values.

**32. (b)** $\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}=\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$

$\Rightarrow\begin{bmatrix}1+4+4&2+2-4&a+4+2b\\2+2-4&4+1+4&2a+2-2b\\a+4+2b&2a+2-2b&a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$

$\Rightarrow a+4+2b=0 \Rightarrow a+2b=-4$ ...(i)
$2a+2-2b=0 \Rightarrow 2a-2b=-2$
$\Rightarrow a-b=-1$ ...(ii)
On solving (i) and (ii) we get
$-1+b+2b=-4$ ...(i)
$b=-1$ and $a=-2$
$(a, b)=(-2,-1)$

**33. (b)** For trivial solution,

$\begin{vmatrix}1&\lambda&-1\\\lambda&-1&-1\\1&1&-\lambda\end{vmatrix}=0$

$\Rightarrow -\lambda(\lambda+1)(\lambda-1)=0$
$\Rightarrow \lambda=0,+1,-1$

**34. (d)** $A(\text{adj }A)=A\,A^T$

$\Rightarrow A^{-1}A\,(\text{adj }A)=A^{-1}A\,A^T$

$\text{adj }A=A^T$
$\Rightarrow\begin{bmatrix}2&b\\-3&5a\end{bmatrix}=\begin{bmatrix}5a&3\\-b&2\end{bmatrix}$
$\Rightarrow a=\frac{2}{5}$ and $b=3$

$\Rightarrow 5a+b=5$